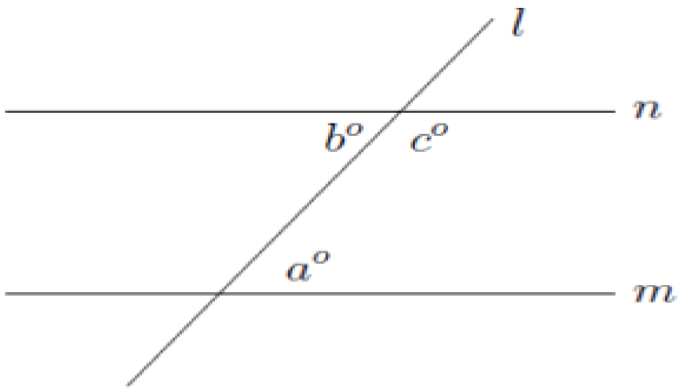
Digital SAT Math Hard Problems With Solution

Desmos

By **Sofian Makhlouf**

1.



In the figure, lines m and n are parallel.

If $a = 5x - 35$ and $b = 3x + 5$, what is the value of c?

A) 20

B) 65

C) 115

D) 135

2.

$$f(x) = (x - 3)(x + 5)$$

For what value of x does $f(x)$ reach its minimum?

A) 0

B) -1

C) -5

D) -16

3.

The density of gold is $19{,}320$ kilograms per cubic meters.

A given solid gold cube has a mass of 154.56 kilograms. What is the length, in meters, of one edge of this cube?

A) 0.2

B) 0.3

C) 0.4

D) 0.5

4.

A company produces two types of items: Model X and Model Y. Each Model X requires 2 hours of assembly time, and each Model Y requires 5 hours. The total available assembly time is at least to 40 hours. The company must produce at most 15 items.

What is the minimum number of items of Type Y the company can produce?

A) 2

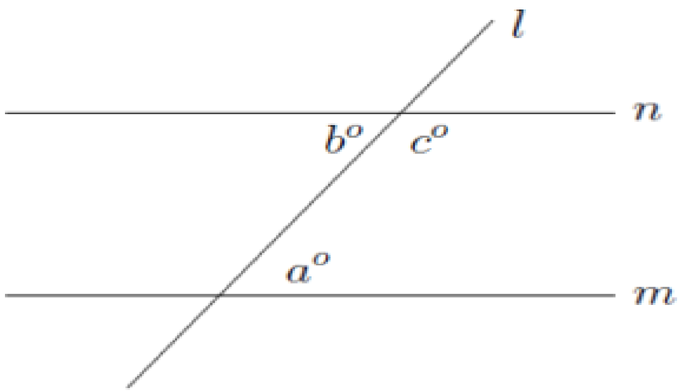
B) 3

C) 4

D) 5

Solution

1.



In the figure, lines m and n are parallel.

If $a = 5x - 35$ and $b = 3x + 5$, what is the value of c?

A) 20

B) 65

C) 115

D) 135

Solution: correct choice is C)

lines m and n are parallel. Therefore, $a = b$.

Therefore, $5x - 35 = 3x + 5$.

Add 35 to both sides, we get $5x = 3x + 40$.

Subtract $3x$ from both sides, we get $2x = 40$.

Divide both sides by 2, we get $x = 20$

Therefore, $b = 3x + 5 = 3 \times 20 + 5 = 65$

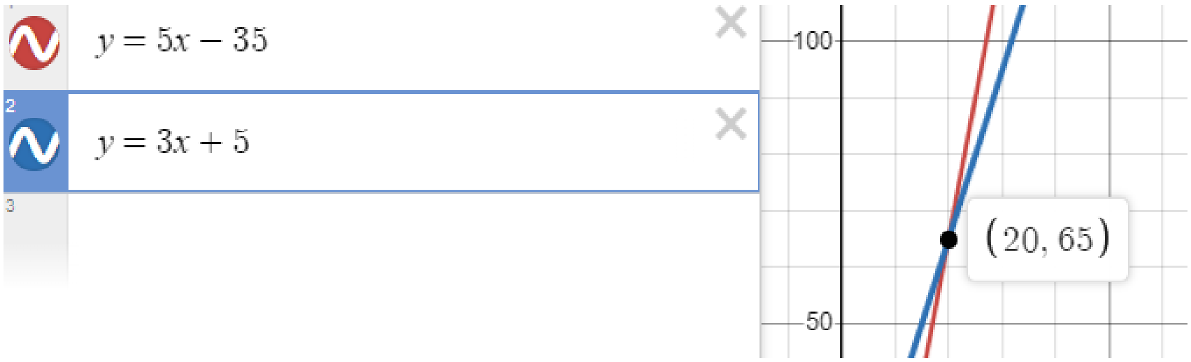
We have $b + c = 180$

Therefore, $c = 180 - b = 180 - 65 = 115$

Second method (Desmos)

lines m and n are parallel. Therefore, $a = b$.

Therefore, $5x - 35 = 3x + 5$.



By Desmos, for $x = 20$, we have $a = b = 65$

We have $b + c = 180$

Therefore, $c = 180 - b = 180 - 65 = 115$

2.

$$f(x) = (x-3)(x+5)$$

For what value of x does $f(x)$ reach its minimum?

A) 0

B) -1

C) -5

D) -16

Solution: correct choice is B)

First method

We have

$$f(x) = (x-3)(x+5) = x^2 + 2x - 15 = ax^2 + bx + c$$

Where $a = 1$, $b = 2$ and $c = -15$

So, $f(x)$ reach its minimum at $x = \frac{-b}{2a} = \frac{-2}{2} = -1$

Second method

We have

$$f(x) = (x-3)(x+5)$$
$$f(x) = x^2 + 2x - 15$$
$$f(x) = x^2 + 2 \times x \times 1 + 1^2 - 1 - 15$$
$$f(x) = (x+1)^2 - 16$$

Therefore,

$$f(x) \geq -16 = f(-1)$$

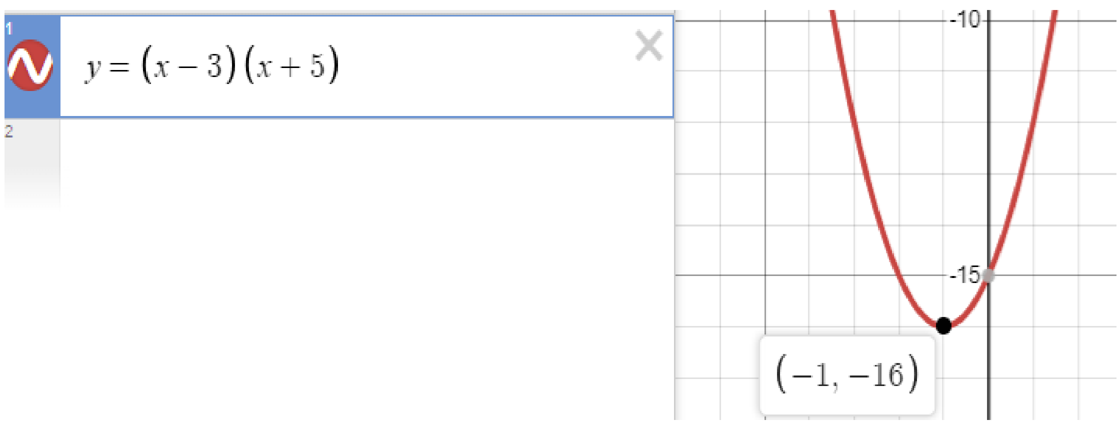
Therefore, $f(x)$ reach its minimum at $x = -1$.

Or we have the vertex expression:

$$f(x) = (x+1)^2 - 16 = a(x-h)^2 + k$$

Therefore, $f(x)$ reach its minimum at $h = -1$.

Third method (Desmos)



The vertex of the given quadratic function is at $(1, -16)$

Therefore, $f(x)$ reach its minimum at $h = -1$ **.**

3.

The density of gold is $19{,}320$ kilograms per cubic meters.
A given solid gold cube has a mass of 154.56 kilograms.
What is the length, in meters, of one edge of this cube?

A) 0.2

B) 0.3

C) 0.4

D) 0.5

Solution: correct choice is A)

First method

Denote by a the length in meters of the edge of the given cube.
Therefore, its volume is $V = a^3$ in m^3
Its mass is $M = 154.56$ in kg
The density $D = 19{,}320$ is (**constant**) in kilograms per cubic meters, that's to say in $kg \times m^{-3} = \frac{kg}{m^3}$
The density is $D = \frac{M}{V}$. Multiply both sides by V, we get $DV = M$
Divide both sides by D, we get $V = \frac{M}{D}$
We have $M = 154.56$ in kg, $D = 19{,}320$ in $kg \times m^{-3}$
and $V = a^3$ in m^3. Therefore,

$$a^3 = \frac{154.56}{19{,}320}$$

$$a^3 = 0.008$$

$$a^3 = (0.2)^3$$

Therefore, $a = 0.2$ meters.

Second method (Desmos)

Denote by x the length in meters of the edge of the given cube.

Therefore, its volume is $V = x^3$ in m^3

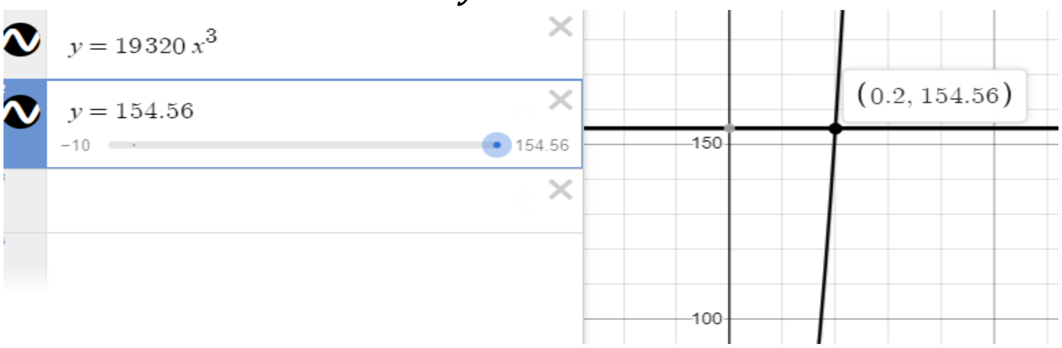
Its mass is $y = 154.56$ in kg

The density $D = 19{,}320$ is (**constant**) in kilograms per cubic meters.

The mass in kilograms is $y = DV$. Therefore, $y = 19{,}320x^3$

Therefore, x is the solution of the system:

$$y = 19{,}320x^3$$
$$y = 154.56$$



By Desmos $x = 0.2$

4.

A company produces two types of items: Model X and Model Y. Each Model X requires 2 hours of assembly time, and each Model Y requires 5 hours. The total available assembly time is at least to 40 hours. The company must produce at most 15 items.

What is the minimum number of items of Type Y the company can produce?

A) 2

B) 3

C) 4

D) 5

Solution: correct choice is C)

Denote by x the number of products Model X
And by y the number of products Model Y
Each Model X requires 2 hours of assembly time, and each Model Y requires 5 hours. Therefore, the assembly time for x products Model X and y products Model Y is $2x + 5y$.
The total available assembly time is at least to 40 hours.
Therefore,

$$2x + 5y \geq 40$$

The company must produce at most 15 items. Therefore,

$$x + y \leq 15$$

The question is: "What is the minimum number of items of Type Y the company can produce?"
That is to say; "What is the minimum value of y in the system of inequalities below?"

$$2x + 5y \geq 40$$
$$x + y \leq 15$$

First method

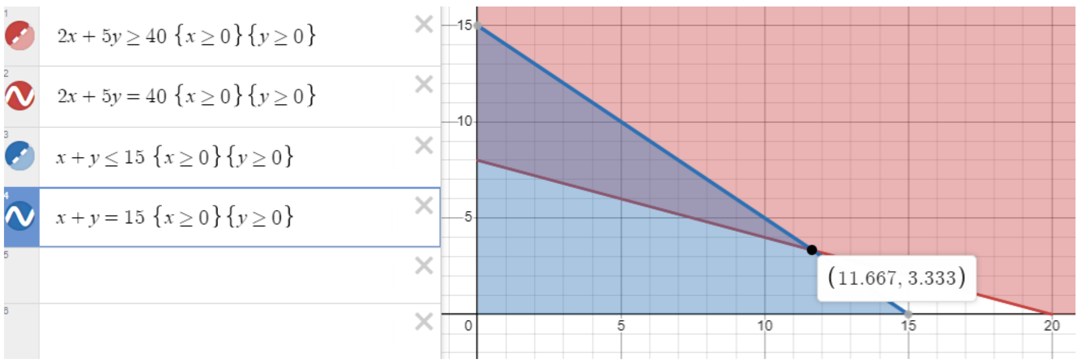
In the second inequality $x + y \leq 15$
Subtract y from both sides, we get $x \leq 15 - y$
Multiply both sides by 2, we get $2x \leq 30 - 2y$
Add $5y$ to both sides, we get $2x + 5y \leq 30 + 3y$
This inequality can be combined with the first inequality

$$2x + 5y \geq 40$$

We get $30 + 3y \geq 40$
Subtract 30 from both sides, we get $3y \geq 10$
Divide both sides by 3, we get $y \geq 3.3333$
Since y is an integer, we get $y \geq 4$
Therefore, the minimum value of y is 4.

Second method (Desmos)

- Draw in red, with $x \geq 0$ and $y \geq 0$ the graph $2x + 5y = 40$, and the region $2x + 5y \geq 40$
- Draw in blue, with $x \geq 0$ and $y \geq 0$ the graph $x + y = 15$, and the region $x + y \leq 15$



The region in purple satisfies both inequalities of the system:

The two graphs intercept at the points $(11.67, 3.333)$, which is the lowest point in the region in purple. So, for all (x, y) solution of our system, we have $y \geq 3.33$. Since y is an integer, we get $y \geq 4$

Therefore, the minimum value of y is 4.

The Key

1.	2.	3.	4.
C)	**B)**	**A)**	**C)**

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